LIFE SAFETY AND SECURITY

Health Safety

Mini review Paper

Mathematical methodologies for the analysis and explanation of articulated and interconnected natural and man-made phenomena

Antonio Gallo, Gaetano La Bella

CORRESPONDANCE: Antonio Gallo
e-mail: antoniogallo@iemest.eu
Phone number: +39091333913

Received: September 18, 2020
Revised: September 23, 2020
Accepted: October 13, 2020.

Abstract

Mathematics is not merely a formal language, but an objective and critical tool for the analysis, interpretation and prediction of a variety of phenomena, both natural and man-made.

This contribution aims to define the value of applied mathematics in the field of scientific analysis, focusing on three fundamental mathematical methodologies: the derivative, the integral and the Fourier transform.

Each of these methodologies allows complex phenomena to be analysed through the analysis of particular trends and the decomposition into a summation of simpler functions, facilitating their interpretation and their complex, multidimensional and difficult to deal with as a whole.

Specifically, the derivative constitutes the cornerstone of infinitesimal analysis, the integral, on the other hand, defines the accumulation and summation of infinitesimal quantities, making it possible to derive overall information from the original data. Finally, the Fourier transform ùùconstitutes an indispensable tool for the frequency analysis of complex and variable systems.

For each of these tools, a theoretical discussion has been provided accompanied by their use in practical activities, in order to document how their use can simplify and clarify the interpretation of complex hermetic phenomena.

Introduction

Mathematical analysis constitutes one of the foundations of science, as it offers a rigorous and multifaceted framework to quantitatively and qualitatively represent a wide range of complex phenomena. Moreover, its application in the translation of empirical investigations into predictive models makes it an irreplaceable tool for the development of scientific knowledge.

Indeed, thanks to advanced analytical techniques such as the derivative, integral and Fourier transform, it is possible to model the dynamics of complex systems, identify existing causal relationships and predict the evolution of phenomena with levels of precision and reliability that depend on the datasets used.

These tools make it possible to analyse and define the multidimensional and often non-linear nature of phenomena, favouring a systematic treatment.

The derivative, for example, represents a fundamental element for analysing the local behaviour of a given function as the variables of the observed phenomenon vary instantaneously, showing how these variations reciprocally condition the overall behaviour of the phenomenon.

In Physics, this tool finds application in the study of the variation of velocity and acceleration, while in Geology and Chemistry it allows one to examine phenomena governed by temporal or spatial dependencies, such as seismic waves or the diffusion of substances in a certain solution.

At the same time, the integral plays a substantial role in the analysis of cumulative quantities, thanks to the sum of infinite infinitesimal contributions that make it possible to obtain the representative function of the phenomenon studied.

In the field of Statistics, the application of the integral function allows the estimation of cumulative probability distributions, while in Climatology it allows the analysis of cumulative trends over extended time intervals, facilitating the understanding of long-lasting or large-scale phenomena.

Finally, the Fourier transform is one of the most powerful and multifaceted tools for analysing complex and often chaotic functions. By breaking down the complex function into its fundamental frequencies, it allows one to identify periodic patterns and isolate significant components otherwise hidden within a global representation. This approach is particularly useful in the processing and analysis of seismic and climatological waves, where the extraction of critical information enables the evaluation of the determining components of the function representing a given phenomenon, otherwise chaotic to the eye.

Finally, if one combines the use of the derivative, the integral and the Fourier transform, it is possible to compose models that go beyond the representation of a given phenomenon, providing clear and comprehensive information represented by the function of the complex phenomenon being studied, making it possible to identify its fundamental components and understand its evolution in highly complex and multidimensional contexts.

The Derivative of a Function

The derivative of a function represents one of the fundamental approaches to analysing and understanding the behaviour of one variable in relation to another, making it possible to quantify the instantaneous variation of dynamic and complex phenomena with respect to a given independent variable.

Mathematically, the derivative of the function f(x) at a point ‘x’ is defined as the limit of the incremental ratio tending to zero of the growth of the independent variable:

$$\boldsymbol{f(x) = \lim_{\Delta x \to 0} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right)}$$

The derivative defines the rapidity with which the function f(x) varies at the point considered with respect to the variable ‘x’, making it possible to observe both the local and global behaviour of a given phenomenon and to understand its evolution. In fact, this tool makes it possible to identify critical characteristics of the function, such as its maxima, minima and points of inflection.

In essence, the derivative enables the study of complex phenomena by describing their temporal or spatial evolution, as well as developing predictive models capable of representing their behaviour in a rigorous manner. This predictive capability is particularly relevant in fields such as physics, geology and climatology, where the phenomena analysed have a strongly dynamic nature. From a purely analytical point of view, the derivative of a function at a certain point represents the slope of the tangent line to the curve of the continuous function at that point. This value reflects the instantaneous variation trend of the function. In other words, if we define a point ‘x’ on a given curve, the derivative f(x) will indicate the ‘speed’ with which f(x)varies as ‘x’ changes.

Consequently, the derivative does not merely provide a measure of the variation trend, but represents an articulated method for analysing dynamic and complex systems.

Finally, through a quantitative approach, it allows us to define the instantaneous changes present in the function of a certain phenomenon, being able to understand their content within a broader context, making it possible to define mathematical models capable of describing complex phenomena with high precision and reliability.

Integral of a Function

The integral of a function represents the inverse operation of the derivative and plays a fundamental role in mathematical analysis, especially in the representation of cumulative quantities and accumulation processes.

Specifically, the definite integral of a function f(x)dx in the interval [a, b] is expressed as:

$$\int_a^b f(x)dx$$

defining, in effect, the area enclosed between the curve of the function f(x) and the ‘x’ axis, limited in the interval [a, b], i.e. the total magnitude associated with a certain variable that varies continuously within a given interval.

The integral represents the infinite sum of infinitesimal quantities, each of which contributes to the calculation of the total magnitude.

This interpretation is fundamental to understanding concepts related to the accumulation and continuous variation of a certain variable within a defined interval, as it allows us to determine cumulative quantities by integrating a continuous distribution of values.

This methodology is used in Physics to calculate quantities such as work, obtained by integrating force along a displacement, or total energy, derived by integrating an energy density over a volume, or in Statistics, to define cumulative probability or to analyse frequency distributions.

Essentially, the integral represents an indispensable tool in the mathematical analysis of complex phenomena, being able to unite local and global aspects of dynamic and complex phenomena through a cumulative representation, allowing a holistic view of the natural phenomena that characterise our universe.

3. Fourier Transform of a Function

The Fourier transform is an extremely powerful mathematical technique that can decompose a representative function of a given parameter, whether periodic or non-periodic, into a combination of sine waves.

This approach allows a complex function to be represented as the sum (or integral) of its fundamental sine-wave components, each characterised by a certain frequency.

For a function f(x)dx, the Fourier transform is formally defined as:

$$\mathbf{F(f)(\omega)} \int_{-\infty}^{+\infty} f(x)\, \mathbf{e^{-i\omega x} dx}$$

where ‘ω’ represents the angular frequency and ‘e-iωx’ describes the complex oscillations associated with each frequency.

The Fourier transform, therefore, determines the contribution of each frequency to the original function by transforming it from the time (or spatial) domain to the frequency domain.

This transformation is essential to be able to distinguish and analyse the distribution of the frequencies present in the original function, making it easier to understand the phenomena that generated them, highlighting dominant frequencies, periodic components and their intensity (amplitude). Furthermore, it makes it possible to visualise how this information is distributed within the initial function, providing, in fact, a clear representation of the 'harmonic' structure and 'oscillations' present in the complex system analysed.

In the case of 'periodic functions', the Fourier transform identifies the main frequencies and their intensity, representing the initial signal as a discrete sum of sinusoidal oscillations.

For 'non-periodic functions', on the other hand, it provides a continuous representation of frequencies, covering the entire spectrum and offering a complete description of the variations present in the signal.

The fact of being able to distinguish and identify 'periodic components' in apparently chaotic signals, showing repetitive patterns, and 'non-periodic signals' defined with continuous representations, defines its relevant usefulness and effectiveness, which have, in fact, made it a mainstay in applied sciences for the study of frequency distribution in wave phenomena, for the analysis of seismic waves, allowing the frequencies of primary and secondary waves to be isolated, which have enabled the reconstruction, in macroseismic studies, of the Earth's internal structure.

The Fourier transform, therefore, is not limited to being just a mathematical tool, but represents an objective and precise method for analysing and understanding complex phenomena, making it possible to highlight indispensable information about the phenomenon being studied, starting from apparently disordered and chaotic data.

Conclusions

Mathematical analysis, using fundamental tools such as the derivative, integral and Fourier transform, is indispensable for interpreting and modelling complex and seemingly chaotic phenomena in science.

The synergistic combination of derivative, integral and Fourier transform offers an extraordinarily powerful mathematical framework for dealing with and modelling complex phenomena. The derivative makes it possible to analyse the instantaneous behaviour of a function, providing a detailed view of local changes; the integral makes it possible to accumulate these changes, outlining a global and cumulative perspective of the analysed phenomenon; finally, the Fourier transform translates these changes from the temporal or spatial domain to the frequency domain, allowing hidden patterns and structures to be identified.

This interaction between mathematical tools is crucial in interdisciplinary applications, such as the study of seismic waves, where the derivative analyses instantaneous changes in seismic wave propagations, the integral calculates the total energy transferred and the Fourier transform decomposes complex signals to isolate dominant frequencies.

Similar approaches also find application in Engineering, Climatology and Biology, demonstrating how the application of these methodologies, whether used singly or in combination, provide a solid basis with which to analyse a complex phenomenon in a scientific, authentic and objective manner, making it possible to solve complex scientific problems, offering accurate predictive models and a deeper understanding of the systems analysed.

The above makes it possible to state that mathematics is not merely a formal language, but represents an essential, precise and objective methodology for exploring complex phenomena, succeeding in predicting and optimising the models that define the multiple fields of study.

The rigorous application of such tools not only allows us to gain a deeper and truer understanding of reality, but also to contribute significantly to improving sustainability and solving global challenges.

Mathematics, capable of integrating theoretical rigour and practical applications, allows complex problems to be analysed with an analytical and oriented approach, capable of preserving human well-being and environmental balance.

Ultimately, mathematical techniques are not just a simple technical means, but represent a lens through which to observe and analytically interpret complex and at first sight chaotic phenomena, defining a scientific, rigorous and real understanding of them.

Bibliography

1. Bracewell, R. N. (2000). *The Fourier Transform and Its Applications*. New York: McGraw-Hill.
2. Courant, R., & John, F. (1999). *Introduction to Calculus and Analysis*. New York: Springer.
3. Debnath, L., & Bhatta, D. (2007). *Integral Transforms and Their Applications*. Boca Raton, FL: CRC Press.
4. Evans, L. C. (2010). *Partial Differential Equations*. Providence, RI: American Mathematical Society.
5. Fourier, J. B. J. (1822). *Théorie Analytique de la Chaleur*. Paris: Firmin Didot.
6. Mallat, S. (2009). *A Wavelet Tour of Signal Processing: The Sparse Way*. San Diego: Academic Press.
7. Papoulis, A. (1984). *Signal Analysis*. New York: McGraw-Hill.
8. Rudin, W. (1976). *Principles of Mathematical Analysis*. New York: McGraw-Hill.
9. Strang, G. (2016). *Introduction to Linear Algebra*. Wellesley, MA: Wellesley-Cambridge Press.
10. Tikhonov, A. N., & Samarskii, A. A. (1990). *Equations of Mathematical Physics*. Berlin: Springer.