



Mini review Paper

ADVANCED MATHEMATICS FOR THE DEFINITION OF LOCAL SEISMIC RESPONSE: REXEL AND STRATA IN A CASE STUDY

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Introduction

The choice of the "design earthquake" is a fundamental challenge for the design of strategic works, since it represents the basis for the construction of safe buildings and infrastructures in seismically active areas. This mathematical process assumes particular relevance for infrastructures with a decisive impact on social, environmental and economic levels, which must maintain their functionality even in the event of significant seismic events.

In relation to this, it appears more relevant than ever to define the "design seismic actions" consisting of the critical seismic characteristics of a given site, such as magnitude, ground acceleration, characteristic period and duration of the event, the determination of which occurs through the use of complex mathematical and physical tools, such as the Fourier transform. This paper illustrates how the use of mathematical methods, with particular

reference to Fourier analysis, are important to extract significant indications from the data relating to reference seismic events in a specific geographical area.

To determine the "seismic actions" it is necessary to define a "Local Seismic Response" (LSR) study consisting of the analysis of the set of changes in amplitude, duration and frequency content that a seismic motion undergoes as it passes through the layers of soil between the basic rock formation (bedrock) and the foundation soils of the work. Seismic motion analyses can be represented in both the "time domain" and the "frequency domain". In the "time domain", the descriptive parameters include the maximum (or peak) value of acceleration, velocity or displacement, and the duration of the event, while in the "frequency domain" the Fourier spectrum (frequencies) is studied.

The quantitative evaluation of the "Local Seismic Response" is based on the comparative analysis of the physical

quantities that characterize the seismic motion at the surface level and at the bedrock. In the “time domain”, the most significant parameter is the “amplification factor”, defined as the ratio between the “peak acceleration” at the surface and that at the bedrock. In the “frequency domain”, the main parameter is the “transfer function”, expressed as $H(f)=F_s(f)/F_r(f)$, which represents the ratio between the Fourier spectrum of the motion at the surface and that at the base. This function, being a ratio between two spectra, is distinct from an “amplitude spectrum” $A(f)$, which provides important information on the filtering effect produced by the soil layers on the seismic motion, which, in fact, produces an increase in the amplitude of the motion for some frequencies and reduces it for others. By analyzing the accelerograms of the seismic events recorded at the surface and at the bedrock, the corresponding “Fourier spectra” can be obtained by means of the “Fast Fourier Transform” (FFT). The ratio between these spectra, i.e. the “deconvolution”, provides the site amplification function. Furthermore, the analysis of the exponential components in the time domain allows to reduce the width of the Cauchy or Lorentz probabilistic distribution lines in the frequency domain, identifying the amplified motion components along the path of the seismic wave from the bedrock to the surface.

This study describes an integrated approach that uses data from the “Istituto Nazionale di Geofisica e Vulcanologia” (INGV) catalogues, the “Rexel” software to identify accelerograms compatible with a target spectrum, and the open-source “STRATA” software for the analysis of local seismic response in one-dimensional contexts. STRATA allows to model the linear propagation of seismic waves and the variations of the dynamic properties of

the ground as a function of the deformations induced by an earthquake.

The integrated approach involves the use of Fourier analysis to characterize the spectral content of the analyzed seismic events, with the aim of determining the key parameters of the “design earthquake”. This is done through the integration of seven reference seismic events, selected for their criticality with respect to the site under consideration. The results offer a useful methodological contribution for designers and researchers of applied seismology.

Recent studies have highlighted the importance of spectral analysis and advanced statistical techniques in seismic modeling. However, uncertainties and margins for improvement persist, especially for the integration methods used between empirical data and theoretical models, especially for complex phenomena such as energy defragmentation and local-scale interactions.

Definition of seismic data of the intervention site

The initial phase of a “Local Seismic Response” study consists in identifying and defining the seismic parameters and fragmentation characteristics of the seismic events that have affected or could affect the site under examination. This step is essential to ensure an accurate assessment of the local seismic behavior and a structural design compliant with the regulations.

In particular, through the consultation of the Parametric Catalogues of Italian Earthquakes (CPTI) and other seismic databases made available by the National Institute of Geophysics and Volcanology (INGV), the seismic events of greatest relevance for the site under study are identified. These catalogues provide fundamental data such as magnitude, epicentral distance, hypocentre

depth and focal mechanism of the recorded events, information that represents the basis for the analysis of seismic parameters.

The regulatory and technical requirements, as specified in the Technical Standards for Construction (NTC 2018), require the use of at least seven accelerograms for each analysis. This minimum number allows to refer to the average values of the results obtained, ensuring statistical representativeness and greater reliability in the interpretation of the data. The accelerograms must be chosen in accordance with the “spectro-compatibility” criteria defined by the regulation, in order to ensure that they accurately reflect the characteristics of the seismic motion expected for the site in question.

The selection of the accelerograms is strictly linked to the disaggregation analysis of the seismic hazard, carried out using the tools and databases provided by INGV. This process allows to determine the “shaking values” associated with specific spectral periods of interest, representative of the seismic events expected in the site under study. These values are necessary to identify the key parameters of the seismic motion, such as magnitude (M) and epicentral distance (d), in relation to the four Operational Limit States (SLO, SLD, SLV, SLC) defined by the national seismic hazard model.

Furthermore, the hazard model also considers the focal mechanism and the depth of the hypocenter, fundamental information to understand the type of seismic stresses that could affect the site. These data are integrated with those relating to the local stratigraphy and the dynamic properties of the soil, in order to calculate the “Local Seismic Response” (RSL) and define useful parameters for seismic design, such as the fundamental period of the site, the amplification factor and the response spectrum of the soil.

In summary, the analysis of seismic data for the intervention site represents an essential phase in seismic design. It allows to combine empirical data with theoretical models, ensuring greater precision in the prediction of seismic effects and a more effective design of strategic works and critical infrastructures.

Selection of spectrum-compatible accelerograms

Once the seismic characteristics of a given site have been defined, the selection of a set of spectrum-compatible accelerograms is carried out using the Rexel calculation code (Iervolino et al., 2013). This calculation code, starting from the “disaggregation” data (expressed in terms of magnitude and epicentral distance from the seismogenic source), allows to identify the accelerograms that satisfy the compatibility requirements with the response spectra prescribed by the Technical Standards for Construction (NTC 2018). These accelerograms represent a fundamental element, since they constitute the seismic input for the definition of the “design earthquake”.

The selection procedure in Rexel is based on the analysis of the seismic data associated with the site and on the identification of the specific contribution to the local seismicity deriving from the identified seismic events. This analysis takes into account the fundamental characteristics of the events, such as magnitude (M) and epicentral distance (d), allowing to select a set of accelerograms representative of the seismic motion expected for the site.

The choice of accelerograms for the design limit states (SLO, SLD, SLV, SLC) is defined according to the statistical criterion of compatibility with the reference normative spectrum. In practice, the set of accelerograms is selected whose mean

spectrum has the lowest standard deviation compared to the target spectrum defined in the NTC. This approach guarantees that the chosen accelerograms accurately represent the seismic behavior expected for the site, reducing the uncertainties due to the variability of the seismic parameters.

The identified accelerograms therefore constitute the seismic input for the intervention site. However, for correct use, they must be scaled to the site-specific acceleration value (a_g), which takes into account the local seismic characteristics and ground conditions.

The analysis of the Rexel code involves the insertion of the design parameters, such as the “Reference Period” (T), the “seismic amplification factor” (S) and the “soil type” according to the NTC classification. Subsequently, the accelerograms are searched through the matching of the target response spectra of the accelerograms present in the seismic catalogues, using minimum deviation criteria between the target spectrum and that of the selected accelerograms.

The selected “set of accelerograms” is essential to realistically simulate the seismic stresses expected for the site, as they constitute the input for nonlinear or linear dynamic analyses, aimed at characterizing the behavior of the structure and the soil during a seismic event. Furthermore, the scaling process to the acceleration value (a_g) ensures that the seismic input corresponds to the local conditions, taking into account the geotechnical and structural peculiarities of the site.

The selection of “spectrum-compatible accelerograms” therefore represents an essential phase in seismic design, since it allows to integrate empirical data with

theoretical models, improving the reliability and effectiveness of design analyses.

Local Seismic Response Modeling

The modeling of the “Local Seismic Response” represents a crucial phase in the seismic analysis, aimed at understanding the interactions between the seismic motion of an earthquake and the stratigraphic characteristics of the design subsoil. This process allows to estimate the amplification of seismic waves at the surface level and the associated effects on the ground motion, providing essential parameters for the definition of the “design earthquake”.

Specifically, seismic modeling determines the seismic amplification of the site, i.e. how seismic waves are amplified or attenuated when crossing the soil layers constituting the vertical of the project site, in addition to identifying the predominant period of the soil, i.e. the resonance frequency, which are extremely critical parameters for the design of works in seismic zones.

Furthermore, through the definition of the site model, it is possible to determine the seismic response spectrum at the surface that takes into account the local soil conditions and therefore evaluate the safety of future works, through the definition of realistic acceleration inputs, of fundamental importance for structural design.

“Local Seismic Response” modeling is generally conducted through “linear-equivalent” approaches, defined by a simplified approach that considers the dynamic properties of the soil as constant, but iteratively modified as a function of the deformation level, and by “advanced nonlinear analyses”, a more realistic approach, which directly simulates the nonlinear behavior of the soil under the action of the seismic input, carried out using a

specific free software for academic purposes, for 1D analysis, STRATA (Seismic Response Analysis of 1D Soil Layers), created by Jonathan P. Stewart and his team at the University of California, Los Angeles (UCLA). The execution of the “Local Seismic Response” modeling with STRATA involves a first phase of definition of the real model of the site, through the definition of the geological, geophysical and geotechnical characteristics of the same, obtained by performing specific field and laboratory investigations and a subsequent phase of interaction of the model created with the accelerograms selected with Rexel, compatible with the regulatory target spectrum. This procedure redefines the shear modulus and the damping coefficient based on the deformation induced by the inserted seismic input, originating a response in terms of accelerations and displacements at the surface level.

In this way it is possible to simulate the propagation of seismic waves through a real stratified profile, defined through the dynamic properties of the soil, managing to define the amplification factor as a function of frequency. The response spectra at the surface and in depth and the deformation and stress profiles for each layer.

“Local Seismic Response” modeling provides a crucial contribution to seismic design. The results obtained allow to define realistic and site-specific seismic inputs, improving the safety and effectiveness of engineering works. The approach also allows to integrate empirical data with theoretical models, providing a deeper understanding of local seismic effects.

Identifying dominant frequencies and energy components through Fourier transform

The application of the “Fourier transform” (TF) plays a fundamental role in the definition of the “Local Seismic Response” (RSL), which is applied to examine the frequency content of the seismic signals, both as input (accelerograms) and as output (surface and bedrock response) derived from the modelling defined with STRATA.

Before using the accelerograms selected with Rexel as input for STRATA, the Fourier transform is applied to define the “spectral content of the signals”, as this mathematical tool is able to identify the dominant frequencies and energy components present in each accelerogram. This step is essential to understand whether the frequency content of the accelerograms is representative of the local geological conditions and compatible with the regulatory target spectrum. Furthermore, its application allows to verify the characteristics of the signal in terms of “frequency amplitude”, in order to ensure that there are no anomalies, disturbances or unrealistic components.

For a function $f(x)dx$, the Fourier transform is formally defined as:

$$\mathbf{F}(\mathbf{f})(\boldsymbol{\omega}) \int_{-\infty}^{+\infty} \mathbf{f}(x) e^{-i\boldsymbol{\omega}x} d\mathbf{x}$$

where “ $\boldsymbol{\omega}$ ” represents the angular frequency, while the term “ $e^{-i\boldsymbol{\omega}x}$ ” describes the complex oscillations associated with each frequency.

The Fourier transform, therefore, determines the contribution of each frequency to the original function, transforming the latter from the temporal (or spatial) domain to the frequency domain. For each earthquake identified as a possible seismic source, spectrograms are then generated through the Fourier transform that highlight the predominant frequencies, their amplitude and temporal variation.

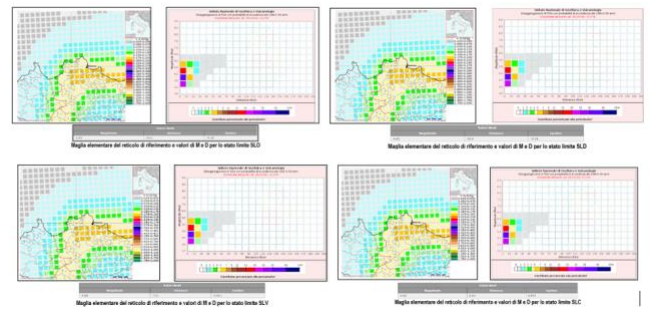
This methodology is also used to analyze the spectral amplification between the base of the

stratigraphic profile and the surface, allowing to quantify the amplification factor, through the comparison between the amplitude of the frequencies between the input and output signals and the characterization of the local amplification, through the identification of the frequencies that are most influenced by the propagation of the waves in the subsurface.

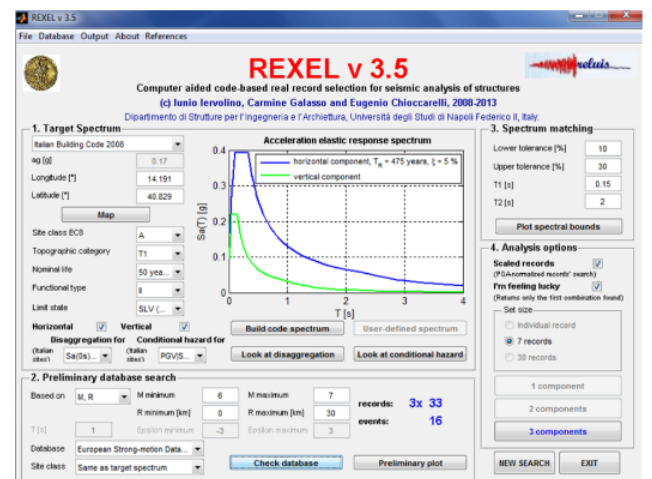
The Case Study: The Hospital of Excellence ISMETT 2

The “Local Seismic Response” (RSL) study for the construction of ISMETT 2, commissioned by the undersigned to UPMC and ISMETT in July 2021, was conducted in the intervention area located in the municipality of Carini. In compliance with the Technical Standards for Construction 2018 (NTC 2018), the Seismic Microzonation Guidelines and Criteria (ICMS, 2008; Castenetto et al., 2014) and Ordinance no. 55 of 24/04/2018 of the Extraordinary Commissioner for reconstruction – 2016 Earthquake, an in-depth study of the geological and tectonic conditions of the site was carried out, accompanied by an extensive campaign of geognostic, geophysical and laboratory analysis investigations, in order to reconstruct a three-dimensional model of the subsoil.

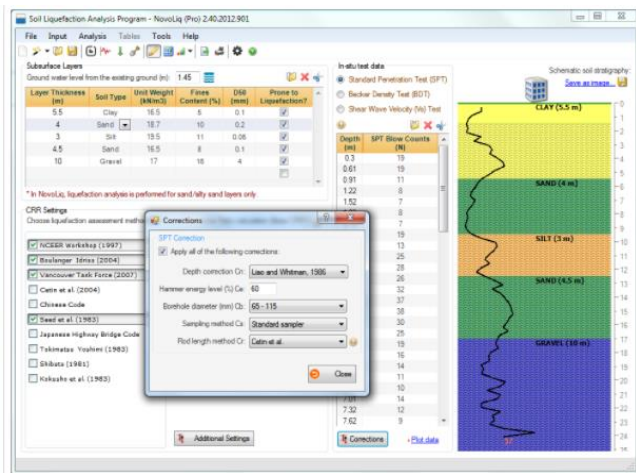
This approach has allowed the design of a strategic and excellent hospital to be prepared, guaranteeing maximum seismic safety. Through the resources provided by the National Institute of Geophysics and Volcanology (INGV), seismic hazard disaggregation analyses were performed, identifying the magnitude (M) and epicentral distance (d) diagrams for the various limit states (SLO, SLD, SLV, SLC).



Subsequently, using the REXEL calculation code, the selection of the combinations of the previously identified spectrum-compatible natural accelerograms with the normative spectra of the NTC 2018, of the Eurocode 8 (EC8-CEN, 2003) and of the ASCE/SEI 7-10 was carried out.

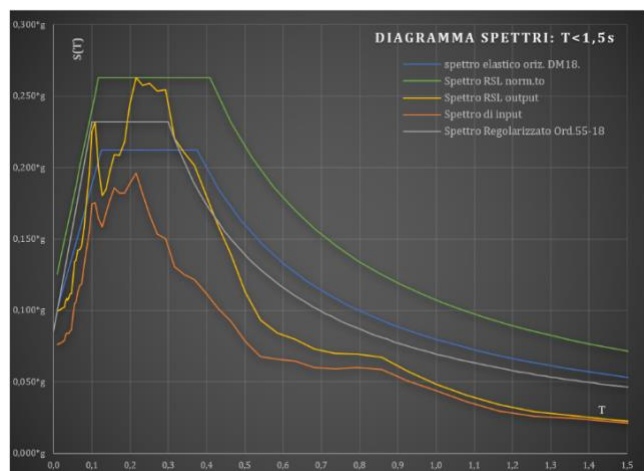


In this phase, in order to define calculation conditions in a model as real as possible, the geognostic and geophysical data acquired during the planned campaigns were integrated, providing for a mathematical modelling of the site under study.



This procedure allowed to evaluate the “Local Seismic Response” through an approach based on the theory of seismic wave propagation in the subsoil and on the theory of nonlinear and dissipative behavior of soils. This analysis allowed to evaluate both the variations in amplitude of the seismic motion, but also the filtering effect exerted by the soil, responsible for the changes in the frequency content of the seismic event.

Before entering and analyzing the selected accelerograms with Rexel, it is necessary to enter the data relating to the seismostratigraphic and elastic properties of the subsoil into STRATA. Once the calculation has started, it is possible to simulate the propagation of the seismic waves in the defined seismostratified model.



This analysis allows to return the site amplification values, the response spectra at

the surface and in the bedrock and the deformation profiles for each layer.

For the Carini site, the elastic response spectra for the various limit states (SLO, SLD, SLV, SLC) were developed, normalized and regularized according to Annex 1 of Ordinance no. 55/2018.

The analysis conducted highlighted a “residual error” lower than 2% for all limit states, confirming the high reliability of the adopted model.

The “normalization” procedure applied, based on the ICMS method (Pergalani and Compagnoni, 2013), was preferred for its statistical coherence and normative reference. This method uses specific formulas for the calculation of amplification factors, ensuring greater representativeness of local response spectra compared to average site conditions.

Finally, the regularization of the spectra allowed to transform the results of the numerical simulations into standardized spectra, in line with the simplified forms proposed in the literature (Newmark and Hall, 1982; Romeo, 2007; Liberatore and Pagliaroli, 2014).

The entire process produced a reliable model for the definition of the design earthquake, ensuring realistic and specific seismic inputs for the structural design of ISMETT 2, a work of strategic importance for the territory.

Conclusions

The study conducted highlights how the integration of advanced methodologies based on complex mathematical tools, such as the REXEL and STRATA calculation codes, represents a significant evolution compared to approaches based exclusively on regulatory standards. These tools allow to define with greater detail and precision the seismic conditions of a specific site, contributing to the

determination of the "design earthquake" with real and representative properties of the local context.

This methodological approach, which goes beyond mere standardized prescriptions, allows for a more reliable assessment of seismic amplification and local effects, providing accurate seismic inputs for structural and infrastructural design, particularly for works of strategic importance. The obtained results demonstrate the importance of an interdisciplinary approach capable of combining empirical data, derived from geophysical and geotechnical survey campaigns, with complex theoretical models. This integration not only improves the representativeness of seismic response models, but also allows to address the complexity of seismic design with rigorous mathematical control.

Modeling based on nonlinear dynamic analyses, the adoption of spectrum-compatible response spectra and the definition of transfer functions in the frequency domain ensure a thorough understanding of the filtering and amplification effects exerted by layered soils, which are critical to reduce design uncertainties and increase the safety level of structures.

Finally, the described approach confirms the need to use advanced analytical tools, both to improve the accuracy in simulating local seismic effects and to ensure resilient design, capable of adequately responding to seismic events even of high magnitude.

These methods provide a solid scientific basis for designers and researchers, contributing to a safer, more sustainable structural planning that complies with the needs of land and society protection.

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